REMARKS CONCERNING THE PAPER "ON SOME OPTIMUM ROCKET TRAJECTORIES" BY G. LEITMANN

(ZAMECHANIE K RABOTE G. LEITMANNA "OB OPTIMAL'NYKH TRAEKTORIIAKH RAKET")

PMM Vol. 26, No. 2, 1962, p. 391

A. I. LUR'E
(Leningrad)

(Received January 2, 1962)

In the indicated paper, published in *PMM* 1961, Vol. 25, No. 6, pp. 1475-1481, an important feature is not revealed, robbing the last section ("Solution", p. 1480) of the presupposed significance.

The author did not notice that the projection of the gravity force on the horizontal plane, in which the mass center of the rocket is moving, maintains a constant direction in this plane, i.e. the angle $\varphi + \gamma$ remains constant. This can be easily verified, considering the first and the second of Equations (7), the first and the second of Equations (8) and Formulas (14). Indeed, differentiating the first relation (14) and using the second, we have

$$V\lambda_V\,(\lambda_\gamma^2 + V^2\lambda_V^2)^{-1/2}\,\dot\varphi = -\,\lambda_\gamma V\lambda_V\,(\lambda_\gamma^2 + V^2\lambda_V^2)^{-3/2}\,(V\dot\lambda_V + \dot V\lambda_V)$$

or

$$\dot\varphi = -\frac{\lambda_\gamma}{\lambda_\gamma^2 + \lambda_V^2 V^2}\sqrt{\left(\frac{c\beta}{m}\right)^2 - g^2}\left(\frac{\lambda_\gamma}{V}\sin\varphi + \lambda_V\cos\varphi\right)$$

Using (14) again we obtain

$$\dot\varphi = \mp\frac{\lambda_\gamma V^{-1}}{\sqrt{\lambda_\gamma^2 + \lambda_V^2 V^2}}\sqrt{\left(\frac{c\beta}{m}\right)^2 - g^2} = -\frac{\sin\varphi}{V}\sqrt{\left(\frac{c\beta}{m}\right)^2 - g^2} = -\dot\gamma$$

Thus

$$\dot\varphi + \dot\gamma = 0, \qquad \varphi + \gamma = \text{const} \tag{1}$$

as required.

The equation of motion of the mass center of the rocket in vector notation is of the form

$$\dot{\mathbf{v}} = \sqrt{\frac{c^2\beta_{\max}{}^2}{(m_i - \beta_{\max}\,t)^2} - g^2}\,\mathbf{e}, \qquad \mathbf{e}\cdot\mathbf{e} = 1 \tag{2}$$

where $\mathbf{e}$ is a unit vector of constant direction. From this we obtain

$$\mathbf{v}(t_f) = \mathbf{v}(t_i) + \mathbf{e}\int_{t_i}^{t_f}\sqrt{\frac{c^2\beta_{\max}{}^2}{(m_i - \beta_{\max}t)^2} - g^2}\,dt \tag{3}$$

From this, using the given initial $\mathbf{v}(t_i)$ and final $\mathbf{v}(t_f)$ values of the velocity vector, $\mathbf{e}$ is determined, as well as the minimum time $t_f - t_i$ (or the maximum final mass m_f).

It seems to me that the initial formulation of the differential equations of motion in "natural" form is unfortunate (i.e. using projections along the tangent and the principal normal), because it renders the uncovering of the property (1) of the sought optimum motion more difficult. This property can be easily detected starting from the equation of motion in the form

$$\dot{\mathbf{v}} = \sqrt{\left(\frac{c\beta}{m}\right)^2 - g^2}\,\mathbf{e}, \qquad \mathbf{e}\cdot\mathbf{e} = 1$$

and introducing in solving the variational problem the Lagrange vector $\boldsymbol{\lambda}_v$.

Translated by G.H.